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# Deterministic Teleportation and Universal Computation Without Particle Exchange

Hatim Salih,<sup>1,2,\*</sup> Jonte R. Hance,<sup>1</sup> Will McCutcheon,<sup>1,3</sup> Terry Rudolph,<sup>4</sup> and John Rarity<sup>1</sup>

<sup>1</sup>*Quantum Engineering Technology Laboratory, Department of Electrical and Electronic Engineering, University of Bristol, Woodland Road, Bristol, BS8 1UB, UK*

<sup>2</sup>*Quantum Technology Enterprise Centre, HH Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol, BS8 1TL, UK*

<sup>3</sup>*Institute of Photonics and Quantum Science, School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh, EH14 4AS, UK*

<sup>4</sup>*Department of Physics, Imperial College London, Prince Consort Road, London SW7 2AZ, United Kingdom*

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Teleportation is a cornerstone of quantum technologies, and has played a key role in the development of quantum information theory. Pushing the limits of teleportation is therefore of particular importance. Here, we apply a different aspect of quantum weirdness to teleportation—namely exchange-free computation at a distance. The controlled-phase universal gate we propose, where no particles are exchanged between control and target, allows complete Bell detection among two remote parties, and is experimentally feasible. Our teleportation-with-a-twist, which we extend to telecloning, then requires no pre-shared entanglement between sender and receiver, nor classical communication, with the teleported state gradually appearing at its destination.

In the popular imagination, teleportation has come to refer to the process by which a body or an object is transported from one place to another without taking the actual journey. While a staple of science fiction, science by contrast seemed to rule it out based on the uncertainty principle, which placed a fundamental limit on the accuracy of measurement [1]. No wonder when in 1993 Bennett and colleagues proposed the first quantum teleportation protocol [2], it was soon recognised as a seminal moment in physics. Relying on the non-classical resource of pre-shared entanglement between the communicating parties, an unknown quantum state of a physical system is measured by the sender in such a way allowing its reconstruction at the receiver, while leaving behind its physical constituents. Classical communication is typically required to complete this disembodied transport.

Not only has quantum teleportation become a backbone of quantum technologies such as quantum communication, quantum computing, and quantum networks, it has also played a crucial role in the development of formal quantum information theory. In this respect, pushing the limits of quantum teleportation is of significant importance, which is what we intend to do here by invoking yet another aspect of quantum “weirdness”: exchange-free computation at a distance.

In exchange-free communication, also known as counterfactual communication, a classical message is sent by means of quantum processes without the communicating parties exchanging any particles. With its roots in the phenomena of interactions-free measurement and the quantum Zeno effect [3–8], the first such deterministic protocol was proposed by Salih et al [9], before being experimentally demonstrated by Pan and colleagues [10].

This was generalised to sending quantum information exchange-free for the first time in 2014 by Salih [11],

proposing an exchange-free quantum CNOT gate as a new computing primitive. The exchange-free CNOT was later employed by Zaman et al to propose exchange-free Bell analysis, albeit with a 50% theoretical efficiency limit [12].

By contrast, the controlled  $\hat{R}_z$ -rotation we propose here, based on the above-mentioned CNOT gate, is universal and has no theoretical limit on efficiency. We then combine quantum teleportation with exchange-free computation at a distance, for the first time, to propose deterministic exchange-free teleportation.

Interestingly, a once heated debate over whether exchange-free communication was permitted by the laws of physics (for both bit values) seems now to be resolving; Nature does allow exchange-free communication, and consequently computation at a distance [13–19].

We first go through the chained quantum Zeno effect (CQZE) unit, as given in Fig.1. This is based on Salih’s exchange-free CNOT gate, which has Bob enacting a superposition of blocking and not blocking his side of the communication channel [11, 20]. Switchable mirror SM1 is first switched off to allow the photon into what we call the outer interferometer, before being switched on again. Switchable polarisation rotator SPR1 rotates the photon’s polarisation from  $H$  to  $V$ , by a small angle  $\frac{\pi}{2M}$ :

$$\begin{aligned} |H\rangle &\rightarrow \cos \frac{\pi}{2M} |H\rangle + \sin \frac{\pi}{2M} |V\rangle \\ |V\rangle &\rightarrow \cos \frac{\pi}{2M} |V\rangle - \sin \frac{\pi}{2M} |H\rangle \end{aligned} \quad (1)$$

Polarising beam-splitter PBS2 passes the  $H$  part towards the bottom mirror while reflecting the small  $V$  part towards the what we call the inner interferometer. Switchable mirror SM2 is then switched off to allow the  $V$  part into the inner interferometer, before being switched

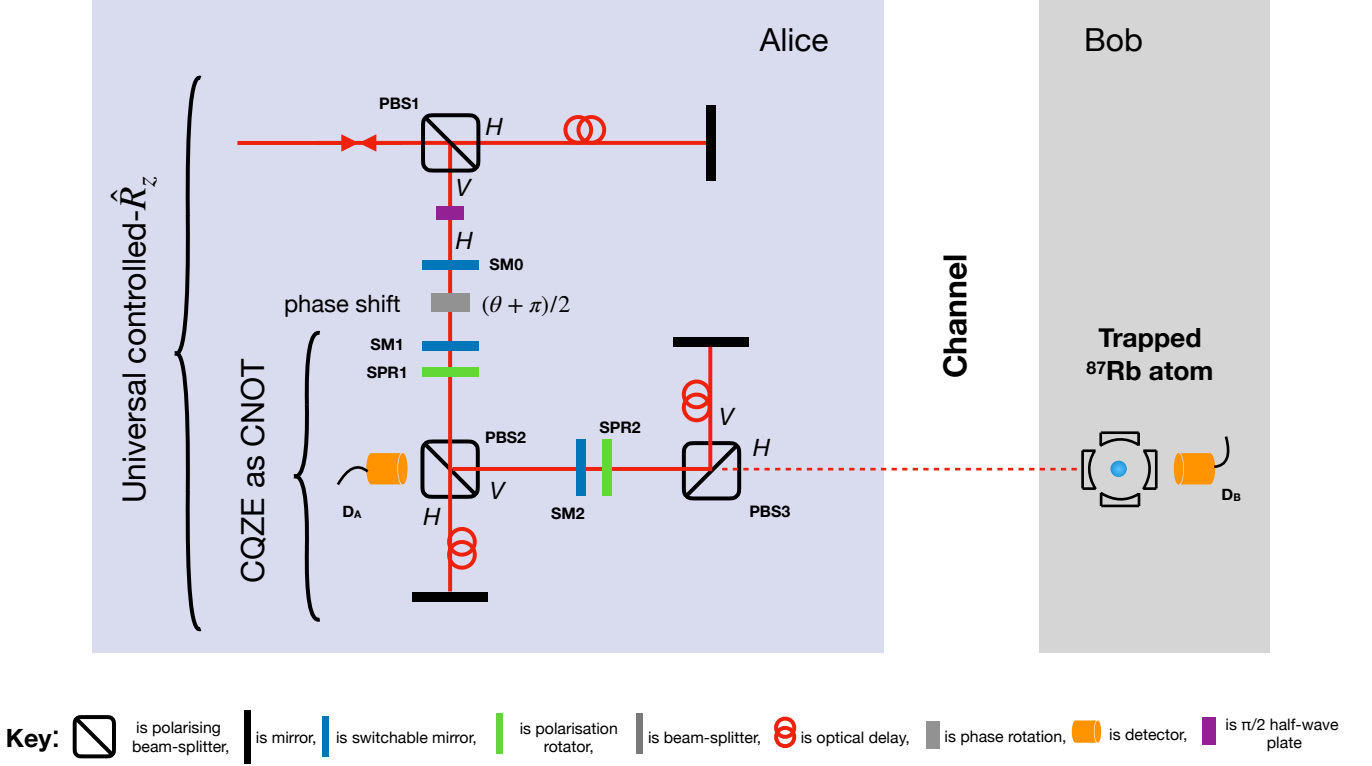


FIG. 1. Our setup for an experimentally feasible, exchange-free controlled- $\hat{R}_z$ , universal gate. This is based on Salih's exchange-free CNOT gate which has Bob enacting a superposition of blocking and not blocking the communication channel by means of a trapped atom [11, 20]. With the addition of a phase-shift plate applying a  $(\theta + \pi)/2$  rotation, switchable mirror SM0, a  $\pi/2$  half wave plate that flips polarisation, polarising beamsplitter PBS1, and an optical delay loop, the chained quantum Zeno effect unit becomes the basis of our controlled phase-rotation universal gate, entangling the states of Alice's photonic qubit and Bob's trapped atom qubit.

on again. Switchable polarisation rotator SPR2 rotates the  $V$  part by a small angle  $\frac{\pi}{2N}$ :

$$|V\rangle \rightarrow \cos \frac{\pi}{2N} |V\rangle - \sin \frac{\pi}{2N} |H\rangle \quad (2)$$

Polarising beamsplitter PBS3 then reflects the  $V$  part towards the top mirror while passing the  $H$  part towards Bob, who is implementing a superposition,  $\alpha |0\rangle + \beta |1\rangle$ , of reflecting back any photon, and blocking the channel, respectively. Specifically, inside the inner interferometer, assuming the photon is not lost to Bob's detector  $D_B$ ,

$$|V\rangle (\alpha |0\rangle + \beta |1\rangle) \rightarrow \alpha (\cos \frac{\pi}{2N} |V\rangle - \sin \frac{\pi}{2N} |H\rangle) |0\rangle + \beta \cos \frac{\pi}{2N} |V\rangle |1\rangle \quad (3)$$

This represents one inner cycle. The photonic superposition has now been brought back together by PBS3 towards SM2. After  $N$  such cycles we have,

$$|V\rangle (\alpha |0\rangle + \beta |1\rangle) \rightarrow \alpha |H\rangle |0\rangle + \beta \cos^N \frac{\pi}{2N} |V\rangle |1\rangle \quad (4)$$

Switchable mirror SM2 is then switched off to let the photonic component inside the inner interferometer out. Since for large  $N$ ,  $\cos^N \frac{\pi}{2N}$  approaches 1, we have,

$$|V\rangle (\alpha |0\rangle + \beta |1\rangle) \rightarrow \alpha |H\rangle |0\rangle + \beta |V\rangle |1\rangle \quad (5)$$

Similarly, for the first outer cycle, starting with the photon at SM1 we have, assuming the photon is neither

lost to Alice's detector  $D_A$ , nor to Bob's  $D_B$  inside the inner interferometer,

$$|H\rangle (\alpha|0\rangle + \beta|1\rangle) \rightarrow \alpha \cos \frac{\pi}{2M} |H\rangle |0\rangle + \beta (\cos \frac{\pi}{2M} |H\rangle + \sin \frac{\pi}{2M} |V\rangle) |1\rangle \quad (6)$$

This represents one outer cycle, containing  $N$  inner cycles. The photonic superposition has now been brought back together by PBS2 towards SM1. After  $M$  such cycles we have,

$$|H\rangle (\alpha|0\rangle + \beta|1\rangle) \rightarrow \alpha \cos^M \frac{\pi}{2M} |H\rangle |0\rangle + \beta |V\rangle |1\rangle \quad (7)$$

Since for large  $M$ ,  $\cos^M \frac{\pi}{2M}$  approaches 1, we have,

$$|H\rangle (\alpha|0\rangle + \beta|1\rangle) \rightarrow \alpha |H\rangle |0\rangle + \beta |V\rangle |1\rangle \quad (8)$$

Switchable mirror SM1 is now switched off to let the photon out. Crucially, this last equation describes the action of a quantum CNOT gate with Bob's as the control qubit, acting on Alice's  $H$ -polarised photon.

We now explain the rest of the setup, which uses the CQZE unit to implement a universal, general-input controlled- $\hat{R}_z$  rotation gate (Fig.1). We begin with a superposition state at Alice,  $a|V\rangle + b|H\rangle$ . This is split at PBS1, with the  $H$ -polarised component going into an optical loop, and the  $V$ -polarised component going through a  $\pi/2$  half wave plate flipping its polarisation to  $H$ , before being admitted into the CQZE unit by turning off switchable mirror SM0, before turning it on again. Upon being exiting the CQZE unit, it is reflected back by SM0, having a phase of  $\theta + \pi$  if  $V$ -polarised (0 if  $H$ ) applied to it by the phase shifter, before going through another run of the CQZE unit. This always produces an  $H$ -polarised state, as noted in [21]. Note that the  $\pi$  term in the phase shifter is a correction term. The photonic component now exits through SM0, which is switched off, before being flipping back to  $V$ -polarisation at the  $\pi/2$  half wave plate, having acquired a  $\theta$  phase shift. It then recombines with the  $H$ -polarised component at PBS1.

Given the initial state of the overall system is

$$(a|V\rangle + b|H\rangle) \otimes (\alpha|0\rangle + \beta|1\rangle) \quad (9)$$

and only the  $V$ -polarised component directly "interacts" with the trapped atom, we get the final state

$$a|V\rangle (\alpha|0\rangle + \beta e^{i\theta}|1\rangle) + b|H\rangle (\alpha|0\rangle + \beta|1\rangle) \quad (10)$$

This is an entangled state between Alice's polarisation qubit and Bob's trapped ion qubit: a controlled-phase rotation, with Alice's as the control qubit and Bob's as the target. Due to the symmetry of control and target

qubits for this type of rotation, it can also be factorised as

$$\alpha|0\rangle (a|V\rangle + b|H\rangle) + \beta|1\rangle (ae^{i\theta}|V\rangle + b|H\rangle) \quad (11)$$

the same controlled-phase rotation expressed differently, now with Bob's as the control qubit and Alice's as the target. Taking the special case when  $\theta = \pi$ , we get a controlled-Z gate.

On universality, our exchange-free controlled- $\hat{R}_z$ , as a two-qubit gate, allows efficient implementation of any quantum circuit when combined with local operations. But there's another sense in which it is universal. As explained in the Appendix, this gate can be operated differently, allowing one party with classical action to enact any desired operation on a second party's remote photonic qubit, exchange-free. This classical action can even control a two-qubit gate at the second party, as shown in [22]. Our controlled- $\hat{R}_z$  gate can therefore be thought of as a universal set in its own right.

Bob needs a way to implement a superposition of reflecting, bit "0", and blocking, bit "1", Alice's photon. There are many ways to go about this; however, recent breakthroughs in trapped atoms inside optical cavities[23], such as the demonstration of light-matter quantum logic gates[24, 25], make trapped atoms an obvious choice.

Bob's qubit is a single  $^{87}\text{Rb}$  atom trapped inside a high-finesse optical resonator by a three-dimensional optical lattice [25, 26]. Depending on which of its two internal ground states the  $^{87}\text{Rb}$  atom is in, a photon impinging on the cavity in Fig. 1 will either be reflected as a result of strong coupling, or otherwise enter the cavity on its way towards detector  $D_B$ . For this, it needs to have mirror reflectivities such that a photon entering the cavity exists towards detector  $D_B$ , similar to [27]. By placing the  $^{87}\text{Rb}$  in a superposition of its two ground states, by means of Raman transitions applied through a pair of Raman lasers, Bob implements the desired superposition of reflecting Alice's photon back and blocking it. Note that coherence time for such a system is of  $\mathcal{O}(10^{-4}s)$  [26], with longer times possible. Therefore, if the protocol is completed within  $\mathcal{O}(10^{-5}s)$ , lower-bounded by the  $\mathcal{O}(10^{-9}s)$  switching speed of switchable components, then decoherence effects can be ignored. (Experimental tricks can ensure the correct number of cycles without having to use switchable optical elements, as in, for instance, Cao et al's experimental implementation of Salih et al's 2013 protocol [10].)

We now move to an exchange-free implementation of teleportation. This is based on the quantum teleportation first devised by Bennett et al [2], but recast such that there is no need for previously-shared entanglement between Alice and Bob, nor classical communication between them. Our teleportation scheme is shown in Fig.2.

In this protocol, we have a photon-polarisation qubit at

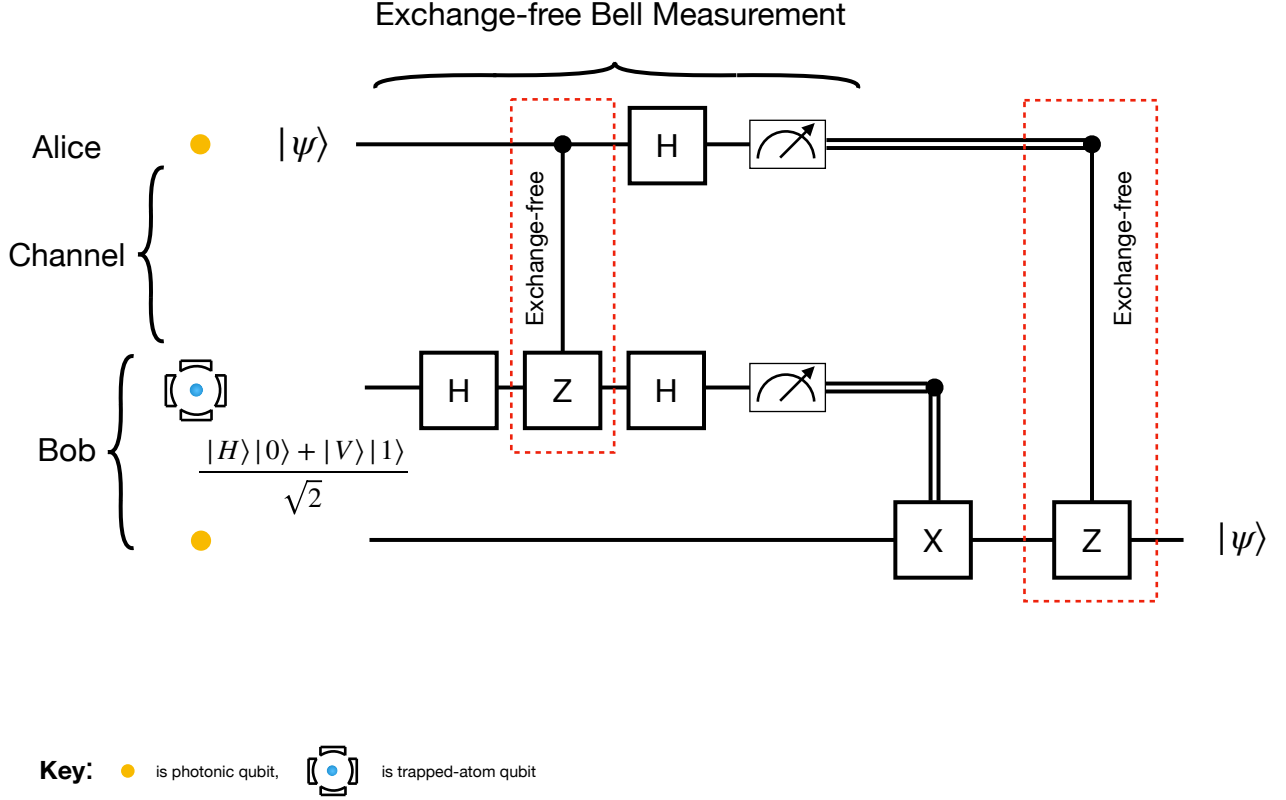


FIG. 2. Our protocol for exchange-free teleportation. In this protocol, Alice has a photon-polarisation qubit, and Bob has a maximally-entangled pair of qubits, one implemented as a trapped-atom enacting a superposition of blocking and not blocking the communication channel, and the other as photon polarisation. Alice's qubit begins in the state to be teleported,  $|\psi\rangle$ . Importantly, complete Bell detection takes place without Alice and Bob exchanging any particles, and instead of classical communication, Alice directly applies a controlled-Z (phase flip) operation on Bob's photonic qubit. The two exchange-free Controlled-Z gates, marked by dashed red-boxes, are instances of the set-up of Fig.1.

Alice, and an entangled pair of qubits, one trapped-atom and the other photon-polarisation, at Bob. Alice's qubit is instantiated in the state to be teleported, e.g. by a third party, while Bob's two modes are in the maximally entangled state

$$\frac{|H\rangle|0\rangle + |V\rangle|1\rangle}{\sqrt{2}} \quad (12)$$

To enact teleportation, Bob first applies a Hadamard gate to his trapped-atom qubit, before Alice applies an exchange-free controlled-Z gate, with her photonic qubit as the control and Bob's trapped-atom qubit as the target. Bob and Alice then apply Hadamard gates onto their respective qubits, before measuring the states in the computational basis for Bob, and in the  $H/V$  basis for

Alice, together performing a complete Bell measurement. Bob then either flips or doesn't flip the polarisation of his photonic qubit based on the classical measurement outcome of his trapped-atom qubit. Alice then, based on the classical measurement outcome of her qubit, either performs an exchange-free controlled-Z on Bob's photonic qubit with her control set to  $|1\rangle$  by blocking both runs, or else sets her control to  $|0\rangle$  by not blocking both runs. These last two steps by Bob and Alice respectively act as the feed-forward step of teleportation (which next-generation trapped atoms are expected to allow) leaving Bob's photonic qubit in the state of Alice's original qubit.

Our exchange-free protocol bears all the hallmarks of teleportation as given by Pirandola et al [28]. Alice's input state is unknown to her, and can be provided by a third party who also verifies the teleported state at Bob.

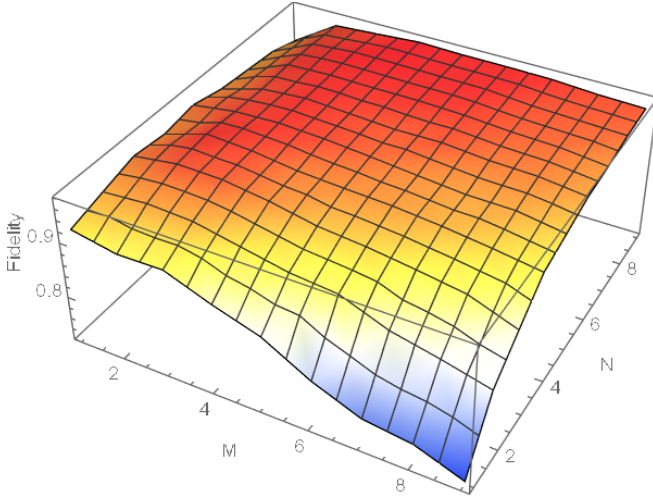


FIG. 3. Average fidelity of our teleportation protocol, shown as a function of the number of outer and inner cycles,  $M$  and  $N$ . This is for an imperfect trapped-atom at Bob that fails to reflect an incident photon 34% of the time when it should reflect, and fails to block the photon 8% of the time when it should block. Fidelity is averaged over 100 points evenly distributed on the Bloch sphere of possible states Alice could send.

The protocol allows complete Bell detection, which in our case is jointly carried out by Alice and Bob exchange-free. The protocol allows the possibility of real-time correction on Bob's photonic qubit, especially with next-generation trapped-atom technology. Lastly, achievable fidelity with our protocol exceeds the  $2/3$  limit of "classical teleportation" which comes from the no-cloning theorem [29]. We give the average fidelity of our protocol in Fig.3, where fidelity  $F(\theta, \phi)$  is

$$F(\theta, \phi) = \langle \Psi_{in} | \Psi_{out} \rangle \langle \Psi_{out} | \Psi_{in} \rangle \quad (13)$$

$\Psi_{in}$  and  $\Psi_{out}$  are the input and output states of the protocol,  $\theta$  and  $\phi$  parameterise the input state's Bloch sphere (azimuthal and radial angle respectively), and the average fidelity is  $F(\theta, \phi)$  averaged over  $\theta$  and  $\phi$  [30]. While at  $M$  of 3 and  $N$  of 10, the average efficiency of the protocol is only 3%, this reaches 30% at  $M$  of 10,  $N$  of 100, quickly tending toward 1.

Speaking of cloning, since quantum telecloning combines approximate cloning with teleportation to transport multiple approximate copies of a states, one would think that our exchange-free teleportation protocol might allow telecloning to be carried out exchange-free. In fact the telecloning scheme of Murao et al [31], which employs a Bell measurement, along with local operations at the receiver based on the Bell detections, can be made exchange-free in a similar manner to how we made teleportation exchange-free. Their scheme starts with an already prepared multipartite entangled state [31], which for our purposes we take to be located at

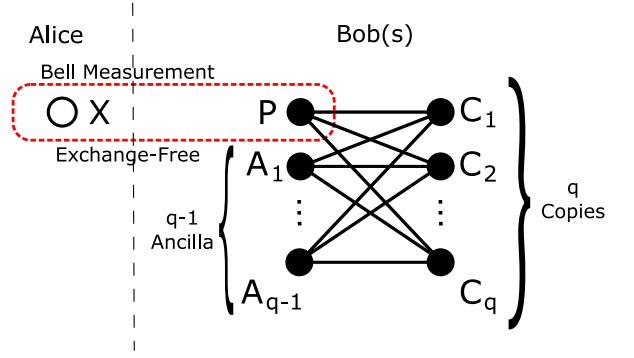


FIG. 4. An entanglement diagram for telecloning using our protocol showing the initial entangled state between the port qubit  $P$ , copy qubits  $C_q$ , and ancilla qubits  $A_{q-1}$ , at Bob(s), and the Bell Measurement done on Bob's port qubit  $P$  and Alice's initial state qubit  $X$ . The thick black lines mark entanglement ( $P$ , and each  $A$  qubit, are entangled with every  $C$  qubit, and each  $C$  qubit is entangled with both  $P$  and every  $A$  qubit), and the rounded box indicates a Bell measurement. Performing this measurement forces the system into one of four states, from each of which (with appropriate feed-forward through  $\sigma_x$  and  $\sigma_z$  rotations on the copy qubits)  $q$  copies of the initial state can be generated.

Bob, with one of the entangled qubits in the form of say a trapped-atom, and the output qubits where the approximate copies appear, all photonic. Alice and Bob jointly perform an exchange-free Bell measurement between Alice's photonic input qubit, and Bob's trapped-atom 'port' qubit, as we show in Fig.4. Based on the classical outcomes of the Bell measurement, Alice applies suitable exchange-free controlled-rotations (Pauli operations) to recover the approximate copies at Bob. The fidelity of these copies is limited by the no-cloning theorem to

$$\gamma = \frac{2q + 1}{3q} \quad (14)$$

where we want to send  $q$  copies of our state. For our protocol, when the trapped-atom interaction is perfect is nil, we always reach this limit of fidelity ( $5/6$  for two copies). In Fig.5, we give the fidelity for an imperfect trapped-atom at Bob that fails to reflect an incident photon 34% of the time when it should reflect, and fails to block the photon 8% of the time when it should block, for different values of  $M$  and  $N$ . While average efficiency for this protocol is 0.2% for  $M$  of 3 and  $N$  of 10, this rises to 14% for  $M$  of 10,  $N$  of 100.

An interesting modification of Salih et al's 2013 protocol was recently proposed by Aharonov and Vaidman, satisfying their criterion, based on weak measurement, for exchange-free communication [18]. Following Salih et al's 2018 paper on counterportation [20], we now show how to implement it in our protocol. In the CQZE module, after applying SPR2 inside the inner interferometer for the  $N$ th cycle, Alice now makes a measurement by

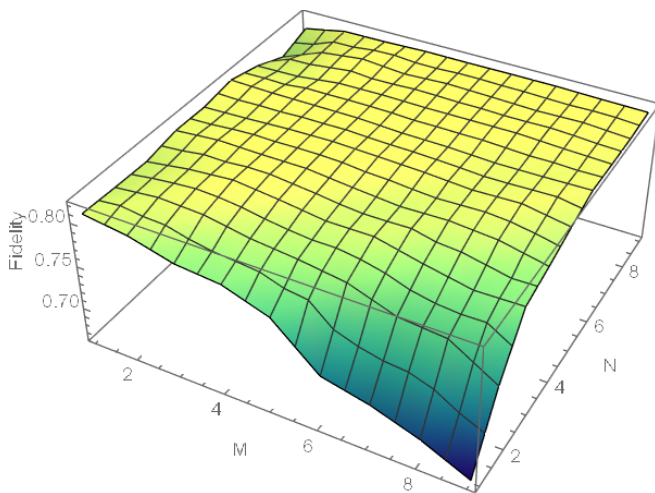


FIG. 5. Average fidelity for telecloning using the controlled-Z gate we give above, plotted for numbers of outer ( $M$ ) and inner ( $N$ ) cycles. This is for an imperfect trapped-atom at Bob that fails to reflect an incident photon 34% of the time when it should reflect, and fails to block the photon 8% of the time when it should block. Fidelity is averaged over 100 points evenly distributed on the Bloch sphere of possible states Alice could send.

blocking the entrance to channel leading to Bob. (She may alternatively flip the polarisation and use a PBS to direct the photonic component away from Bob.) Instead of switching SM2 off, it is kept turned on for a duration corresponding to  $N$  more inner cycles, after which SM2 is switched off as before. One has to compensate for the added time by means of optical delays. The idea here is that, for the case of Bob not blocking, any lingering  $V$  component inside the inner interferometer after  $N$  inner cycles (because of weak measurement or otherwise) will be rotated towards  $H$  over the extra  $N$  inner cycles. This has the effect that, at least as a first order approximation, any weak measurement in the channel leading to Bob is made negligibly small.

We have shown how the chained quantum Zeno effect can be employed to construct an experimentally feasible, exchange-free controlled- $\hat{R}_z$  operation, which is not only a universal gate, but can be considered a universal set. This allowed us to propose a protocol for deterministic teleportation of an unknown quantum state between Alice and Bob, without exchanging particles. The fact that the multiple cycles cause teleportation to happen gradually, in slow-motion so to speak, as opposed to standard quantum teleportation where the teleported state appears at once, is as interesting as it is surprising.

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\* salih.hatim@gmail.com

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### Appendix - Exchange-Free Phase Rotation without Time-Binning

In our recent paper, *Exchange-Free Computation on an Unknown Qubit at a Distance*, we give a protocol that allows Bob to implement any phase on Alice’s qubit, exchange-free [22]. This then forms the basis of a device that we called a phase unit, allowing Bob to apply any arbitrary single-qubit unitary to the qubit, exchange-free. However, an issue that phase unit displayed was that the time the photon exited the device was correlated with the phase applied by Bob. While we provided a way for Bob to undo this time-binning after the fact, it is generally desirable to remove it altogether.

By adapting the controlled phase-rotation above, a phase unit can be constructed that doesn’t exhibit this

time-binning. We use the set-up in Fig.1, but instead with a classical Bob either blocking or not-blocking, and with SM0 keeping Alice’s photon in the device for  $2L$  runs (rather than 2). Bob sets  $\theta$  to  $\pi/L$ , blocking for  $2k$  of the runs and not blocking for  $2(L - k)$ , in units of 2 runs where he either blocks for both or does not block for both. This allows Bob to set a phase on Alice’s photon of  $2\pi k/L$ . We place three of these devices in series, interspersed with a  $-\pi/4$ -aligned Quarter Wave Plate,  $\hat{U}_{QWP\hat{R}_x}(-\pi/2)$ , and its adjoint,  $\hat{U}_{QWP\hat{R}_x}^\dagger(\pi/2)$ , to create a chained- $\hat{R}_z\hat{R}_x\hat{R}_z$  a set of rotations, into which any single qubit unitary can be decomposed. Bob can thus apply any arbitrary single-qubit unitary to Alice’s qubit—both exchange-free, and without time-binning. This also, as we show in [22], allows us to classically control of a universal two-qubit gate, which enables Bob to directly enact in principle any desired algorithm on a remote Alice’s programmable quantum circuit.